Connecting Math and Science for *All* Students

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Hands-on
Contextualized
Highly interactive
Interconnected
Systematic
Engaging
Interesting
Understandable

The words in our little list have a sort of progression. We go from hands-on activities that have context, and we end with engaged students who understand the material we are presenting. What could be better? Sounds easy, right? Well, many teachers may find that they need another word on the list, "systematic," to ensure that *all* students "get" the connections between math and science and improve their understanding.

This article describes a way to provide *systematic* connections between mathematics and science through several hands-on lessons, word problems, and opportunities for problem-solving. The article also describes the benefits to *all* students of linking mathematics and other school subjects.

The 1997 Amendments to IDEA stipulated that students with disabilities have access to and make progress in the general education curriculum.

Special Education Connections

Special education has an extended history of curricular connections. Seguin (1907) commented as far back as 1866: "Teaching facts is not so fruitful as teaching how to find the relations between a single one and its natural properties and connexions" (p. 64). Ingram (1960) and Duncan (1943) suggested that subjects should not be taught alone, but in connections to other subjects. Ingram employed a "unit method" in which students would study a common topic such as "Raising Chickens"; and all skills and processes would be integrated into the topic to ensure that the student recognized the relationships between the subject and its applications.

Other researchers (e.g., Engelmann, Carnine, & Steely, 1991) described how to teach arithmetic computational relationships and their connections within the context of solving word problems in mathematics. Hasselbring and Moore (1996) used contextualized learning environments to teach mathematics to students with learning difficulties. Gersten and Baker (1998) described how to integrate scientific concepts into situated cognitive activities through explicit instruction.

Cawley and Parmar (1993) developed two comprehensive problem-solving programs that integrated mathematics and science within the context of language comprehension and arithmetic. The two programs begin with counting and extend through division and cover a grade span approximating 4 years. The programs are nontraditional in that they engage the student in contextualized problem-solving activities and seek to develop computational proficiency through problem-solving. All activities are "hands-on" and highly interactive. They all take place within science settings (e.g., the matter laboratory; the weather map) and develop proficiency with computation via problem-solving activities.

Making Connections

Our problem-solving programs take advantage of the structure and sequence of math curricula and inject science topics into the lessons. We thus can make interesting and meaningful connections between mathematics and science. This article outlines three mathematics ideas. These are the interrelationships between multiplication and division, ratio, and proportions. The topics cover a grade range of Grades 3 through 8. Each of these topics represents important mathematics principles, as follows:

- The *multiplication and division relationship* is important as a means of describing multiplication and for using the relationship between factors to define division as a search for the missing factor.
- *Ratio* is important as a constant (i.e., pi as derived from circumference/diameter); as a variable for making

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comparisons (e.g., Which is larger/smaller: 2/4, 3/8, or 4/8?); and as a guideline for developing mixtures and solutions (e.g., customized paint orders).
- *Proportions* are important for comparing two ratios and for preparing combinations of products (e.g., If 1 ounce of weedkiller is added to a gallon of water, how many ounces of weed killer must be added to 4 gallons of water?). Many adolescents and adults have difficulty with proportional reasoning (AAAS, 1993), as do students with disabilities (Brownell, Mellard, & Deshler, 1993). Researchers have shown that students with disabilities improve significantly in their understanding and use of proportions under different instructional interventions (Moore & Carnine, 1989).

Our means of developing connections has been the *Activity-Based Mathematics Units.* The sample units in this article integrate science and mathematics at three levels: primary, intermediate, and junior high school. We present the units as a model for activities that you can construct to systematize relationships between mathematics and other subjects. Each lesson includes an introductory or background activity for the topic.

We recommend that the science activities be highly interactive. They should engage the students in a variety of representations and problem-solving activities that give constant meaning to the three mathematics principles.

Multiplication and Division Relationship

Activity-Based Mathematics Unit: Grades 3-5

***Strand*:** Numbers

***Topic*:** Whole Numbers

***Concept*:** Multiplication and Division Relationship

***Focus*:** Direct Applications to Science Problems

***Introduction*:** At this level, students will begin to experience and use many multiplication-division relationships across varying content areas. First, review different combinations of the computational relationships (e.g., 2 x 3 = ? ; ? x 3 = 6; 2 x ? = 6). Next, select a common "hands-on" activity from a content area and conduct the activity. For example, engage the students in an activity to determine the amount of work needed to move an object a specific distance. Provide a measuring tape, a spring scale, and one or more objects. Attach the object to the spring scale until the scale is taut. The number registered on the scale is the *force* on the object. Place the measuring tape on the table and pull the object from one point to another; this is the distance. Make a chart for the data (see Figure 1). Record the observations in the chart.

To determine the amount of work done, multiply the force times the distance. The formula, $W = FD$, represents the multiplication-division relationship in that any missing element can be found by multiplying the factors or by dividing a factor into the product. To determine if the amount of work required to move an object varies under different conditions, repeat the activity by introducing different variables. For example, hang the measuring tape on the wall and pull the object straight up. Change this by pulling the object through sand or, if it floats, across the top of water. Record and compare the results. Call attention to the relationships across each row in the chart (e.g., the force changed, the distance remained the same, the amount of work changed). Modify the activity by moving the same object different distances under the same condition.

As the students develop an understanding of the varying relationships, call their attention to the chart. Erase or mask the data for any one variable (e.g., distance) and ask how the *distance* traveled could be determined by knowing the *force* and the *work*. Do this for a number of combinations, highlighting the multiplication-division relationship.

***Sample Word Problems*:**

1. Takira used a force of 3 pounds to move a chair 4 feet across the kitchen to the counter so she could reach the cookie jar. How much work did she do?
2. Simon did 12 foot-pounds of work to move a bench 4 feet to the foot of the tree so he could reach the first branch and climb the tree. What is the mass of the bench that Simon moved?
3. Nashon did 12 foot-pounds of work to a 3 pound wooden box to build a fort. How far did Nashon move the wooden box?

"Teaching facts is not so fruitful as teaching how to find the relations between a single one and its natural properties and connexions."
—Seguin (1907)

Ratio

Activity-Based Mathematics Unit: Grades 5-6

Figure 1. Force • Distance = Work

Condition	Force	Distance	Work
Horizontal/No Friction			
Horizontal/Friction			
Vertical/No Friction			
Vertical/Friction			
Horizontal/Weight Changes			
Horizontal/Distance Changes			

Strand: Numbers
Topic: Whole Numbers
Concept: Ratio
Focus: Direct Applications to Science Problems

Introduction: *Ratio* is a comparison of two quantities. At this level, students should recognize the relationship of ratio to division (i.e., the ratio of one quantity to another is their quotient). We find the ratio by dividing one number by another; or, as in the present illustration, we seek to identify the "missing factor." It is important that the student comprehend that ratios can represent constants, as in pi, and that ratios can vary for the purpose of making comparisons or for preparing combinations of mixtures or solutions.

Introduce ratio by having the students conduct a variety of activities involving measurement comparisons, such as the following:

1. Measure the length and width of materials within the classroom (e.g., books, desk, or table size). Ask probing questions to focus attention on the relationships between length and width (i.e., they are the same for square regions; they differ for rectangular regions).
2. Mix water and food coloring or water-based paints until the color of the water changes to a specified degree (e.g., light blue; dark blue). Have the students tally the number of units of coloring that is required to attain a certain shade of color.
3. Use a spring scale and move a variety of objects by pulling them with the spring scale. Tally the amount of force needed to move each of the objects.

Researchers have shown that students with disabilities improve significantly in their understanding and use of proportions under interventions like Activity-Based Mathematics Units.

Figure 2. Measurements: Ratio

Length	Width	Ratio
	or	
Width	Length	Ratio
	or	
Color 1	Color 2	Ratio
(Units)	(Units)	
	or	
Object	Force	Ratio
(Mass)	(Pull)	

Make a chart to record the data for each type of measurement (e.g., see Figure 2).

Vary the conditions or combinations of items. Ask the students to record the data on the chart. Inquire as to whether all data are the same (some may be the same—books of the same length and width; others will not—books of different lengths and widths). Take a sample of three or four books of the same length and width and ask the students to multiply the width by the length. Ask the students to compare the answers to each item (i.e., they are all the same). Next, compare three or four books of different lengths and widths in the same way. Again, ask for comparisons (i.e., the answers are all different).

Repeat the activity in different settings (e.g., change from books to the moving of objects; make comparisons in the amount of time different students sleep each day in relation to the length of a day). As the students identify the similarities (e.g., all students who sleep 6 hours per day, sleep one of four 6-hour units) and differences (e.g., those who sleep 8 hours per day, sleep one of three 8-hour units per day).

Work with the students and ask for a way to make a more direct comparison between the amount of time each group sleeps. Ask, "What is common about the day for each?" Answer, "It has 24 hours."

Ask, "What is different about each day?" Answer, "Some sleep 6 hours, and others sleep 8 hours." Suggest that an acceptable way to compare the part of each day that the two groups sleep is to divide the number of hours in each day (a common unit) into the number of hours each group sleeps (different units)(e.g., 6/24 = .25; 8/24 = .33).

Sample Word Problems in Life Science

1. At Farmer Brown's Apple and Pear Grove there are 12 rows of apple trees. The trees in 4 of the rows have a disease. What is the ratio of the rows of trees with disease to rows of trees without disease?
2. Farmer Brown sprays 3 of every 12 trees with organic insect control. She does not spray the other trees. What is the ratio of trees sprayed with insect control to those that are not sprayed?
3. Farmer Brown sprays 3 of every 9 apple trees, but none of the 12 pear trees. What is the ratio of apple trees that are sprayed to the number that are not?

You should develop instructional activities for *ratio* in both real and contrived settings. These settings can focus on measurement, geometry, science, or activities of daily living. In recording and using data, help the students see meaning and relationships in what they are learning.

Proportions

Activity-Based Mathematics Unit: Grades 7-8
***Strand*:** Numbers
***Topic*:** Whole Numbers
***Concept*:** Proportions
***Focus*:** Science Problems

***Introduction*:** *Proportions* are expressions of relationships between two ratios. Students need to understand that the *product of the means equals the product of the extremes*. This principle will be developed through the activities.

Introduce proportions with a variety of activities involving weight/distance relationships with balances of different types (e.g., pan balance, string balance, balance scale). Begin, for example, with a simple activity, in which pairs of students create a balance scale.

Provide the students with a variety of objects of similar and different masses. Instruct the students to use the balance and sort the objects into those with similar and different masses.

Next, have each set of students place two items of equal mass on the balance.

Instruct the students to remove one of the items and to replace it with one having a greater or lesser mass, to create an imbalance. Invite the students to show how they could balance items of unequal mass. Students can then demonstrate that placing the items on opposite ends of the scale and at different distances from the center of the scale will result in a balanced scale. The students can create a chart to record some of these relationships. Begin by having the students determine the mass of each of their objects. Insert the data for mass in the chart. Figure 3 illustrates a completed chart, which you can use to show the students what their chart will resemble when it is finished.

Instruct the students to place the objects at specified distances from the center of the balance, as indicated on the chart. Ask the students to explain their observations for each of the combinations they try. They should identify the principle that *changing either the distance or the mass does not disturb the balance, as long as both sides are changing in the same way.*

Instruct the students to complete the fourth and fifth combinations and to record their observations. Here, mass changes, and distance is held constant. Next, have the students complete the sixth and seventh combinations. Here, mass is constant, and distance changes. At this point, the students should be able to specify the way in which a change in one variable will affect the balance.

Prepare a chart like the one shown in Figure 4 and display it for the students. Indicate that each row has missing data. Instruct the students to gather the appropriate materials and to balance the scale.

If, for example, the student is using apples, the apples should be of the same mass, or very close to each other. The apples could then be hung from a string or put in the pan balance. The mass of the apples could be determined as needed.

Ask each group of students to record their data and observations.

Ask the students if anyone knows an arithmetic way to determine the mass or distance where it is needed without using objects or by changing distances. If someone has a suggestion, work with it to develop the principle of proportions and to write this as fractional representations similar to the following expressions:

Do this for a number of items in different contextual settings over a period of days. Highlight the relationships among Mass 1 and Distance 1 with

Figure 3. Demonstration of Proportional Relationship Between Mass and Distance

	Mass 1	Mass 2	Distance 1	Distance 2
1st Combination	25 gm	25 gm	50 cm	50 cm
2nd Combination	25 gm	25 gm	40 cm	40 cm
3rd Combination	20 gm	20 gm	40 cm	40 cm
4th Combination	30 gm	30 gm	40 cm	40 cm
5th Combination	20 gm	30 gm	40 cm	40 cm
6th Combination	18 gm	23 gm	54 cm	69 cm
7th Combination	15 gm	8 gm	60 cm	32 cm

Figure 4. Mass and Distance Proportional Relationship Problem-Solving

	Mass 1	Mass 2	Distance 1	Distance 2
1st Combination	? gm	25 gm	50 cm	50 cm
2nd Combination	25 gm	25 gm	? cm	40 cm
3rd Combination	20 gm	?gm	40 cm	40cm
4th Combination	25 gm	25 gm	40 cm	? cm
5th Combination	20 gm	?gm	45 cm	45 cm
6th Combination	30 gm	30 gm	40 cm	? cm

What Does the Literature Say About Connecting Math and Science?

Throughout the literature of mathematics and science, many education organizations have described the importance of *connections*. The National Council of Teachers of Mathematics (NCTM, 2000) has indicated that connections foster understanding of mathematics so that *all* students

- Recognize the connections among different mathematics ideas.
- See the relevance of connections between mathematics and other school subjects.

In *Benchmarks for Science Literacy*, the American Association for the Advancement of Science (AAAS, 1993) cited the useful knowledge of science as richly interconnected and asked for students to learn about the interdependence of science, mathematics, and technology. In the proposed National Science Education Standards, the National Research Council (1994) stressed that connecting mathematics and science enhances student understanding of mathematics in the study of science and improves student understanding of mathematics in general. The document *Blueprints for Reform: Science, Mathematics and Technology* (AAAS, 1998) recommended an increased emphasis on the connections between science and other disciplines. All these documents stressed the need to address *all* students.

The 1997 Amendments to the Individuals with Disabilities Education Act (IDEA; Federal Register, 1999) stipulated that students with disabilities have access to and make progress in the general education curriculum. The regulations further defined the "general education curriculum" as essentially the same curriculum presented to students who do not have disabilities. Together, the actions of the major professional organizations, federal law, and the actions currently being undertaken at state levels suggest a need to assure *all* students an opportunity to maximally participate in and make progress in the general education programs of science and mathematics.

those of Mass 2 and Distance 2. For instance, if the product of the means is equal to the product of the extremes then *Mass*1 x *Distance*2 = *Mass*2 x *Distance*1, such that *Mass*1 x 50 = 25 x 50. To solve for the missing factor (*Mass*1) we have 25 x 50 = 1250 and we divide 1250 ÷ 50 = 25, so *Mass*1 = 25.

The programs described here engage the students in contextualized problem-solving activities, through which students develop computational proficiency.

$$\frac{Mass1}{Distance1} = \frac{Mass2}{Distance2} \quad \text{such that}$$

for the 1st Combination $\frac{Mass1}{50} = \frac{25}{50}$

for the 3rd Combination $\frac{20}{40} = \frac{Mass2}{40}$

for the 4th Combination $\frac{25}{40} = \frac{25}{Distance2}$

for the 7th Combination $\frac{30}{Distance2} = \frac{30}{50}$

Sample Word Problems

1. If Jorge and Brian are trying to balance on a seesaw that has seats which can be adjusted in relation to their distance from the center of the seesaw, how far from the center does Jorge who weighs 65 pounds have to be if Brian weighs 40 pounds and is seated 24 inches from the center of the seesaw?
2. At the Old Time Candy Store Hanna decides to buy some gumdrops. On one side of a pan balance, a 12-gram weight is placed 24 centimeters from the center of the pan balance. If the empty pan on the other side is placed 18 centimeters from the center of the balance, how many grams of gumdrops can Hanna place on the empty pan if the two sides are to balance?

Teachers need to explore ways to make accommodations and adaptations to address the needs of their students.

Final Thoughts

Public Law 105-17, popularly referred to as the 1997 Amendments to IDEA, stipulates that students with disabilities are to have access to and make progress in the general education curriculum. It further defines the general education curriculum as *essentially the same curriculum as presented to students who do not have disabilities*. Clearly, the law implies that we must provide students with mild disabilities with more substantive mathematics instruction than we currently offer.

Accordingly, teachers will need to explore ways to make accommodations and adaptations to address the needs of these students. All students will be the beneficiaries as we modify curriculum and instructional methods to accommodate all students and as we redesign assessments to evaluate higher-order outcomes for students in mathematics. The development of comprehensive and systematic material, such as Activity-Based Mathematics Units, could be one solution.

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